INTEGRAL SOLUTION OF 3D ELECTRIC FIELD OF A DISCONNECTOR

P. Karban, M. Donátová

Department of Theory of Electrical Engineering, Faculty of Electrical Engineering, UWB, Univerzitní 26,306 14 Pilsen, Czech Republic, tel.: +420 377634631. E-mail:{ karban, donatm}@kte.zcu.cz

Summary: The disconnectors belong to elements widely used in electrical power engineering and apparatus technology for disconnecting various electric circuits. Usually they work without voltage (the circuit is first switched off by a circuit breaker). Nevertheless, in a fault regime the contacts of the disconnector may carry the full voltage, which may result in the electric arc between them at the moment when the movable contact approaches to the fixed one. In order to estimate this moment it is necessary to know the time evolution of the electric field in the domain between both contacts. This problem is solved in 3D (in somewhat simplified geometry) by the integral technique. The theoretical analysis is supplemented with an illustrative example whose results are discussed.

1. INTRODUCTION

Nowadays, the distribution of electric fields in linear systems containing electrodes carrying prescribed potentials is mostly solved by the differential techniques, prevailingly by the finite element method (see, for example, [1], [2] and [3]). This method is implemented practically in all available professional codes (Flux, OPERA, MagNet and lot of others). It works reliably and the results calculated usually well correspond with the physical reality.

From time to time, however, its employment in more complicated cases is not so advantageous. A typical example is a 3D electric field whose parts move with respect to one another. And when the solved domain is, moreover, large, two serious problems may appear:

- The domain must be remeshed at each time level in accordance with the instantaneous positions of the movable parts.
- Lack of the capacity of the computer necessary for generation of a sufficiently fine discretization mesh (the mesh must remain relatively rough, which may negatively affect the results of computation).

In such cases the integral method may represent a powerful alternative. Of course, its principal ideas are not new. The method is described in numerous references ([4], [5], [6] and others), but its wider applications for larger problems are rather rare, due to several serious lacks listed below:

- The corresponding integral operator (the solution is described by the Fredholm integral equation of the first kind) is not well-posed in the Hadamard sense and sometimes it has to be regularized in order to obtain correct results. But the existing methods of regularization [7], [8] start from the assumption of the knowledge of at least approximate solution and from far they cannot be used for any kernel function.
- The method works with fully or densely populated system matrices. This means that a lot of memory has to be allocated for the corresponding operations and its order (even for good PCs) cannot exceed about 10^4.
- Problems with the evaluation of multiple integrals, particularly when they are improper. On the other hand, their values are always finite both in 2D and 3D.

Nevertheless, some drawbacks of the method may be reduced using special advanced techniques. For example, the number of the degrees of freedom (DOFs) of the task can significantly drop when using the higher-order technique (the distribution of electric field in the elements is supposed not constant or linear as is usual for the classical algorithms, but as a linear combination of more base functions that are of a polynomial character).

The authors have been dealing with the application of various integral techniques for solving selected problems of electromagnetism for several years. They developed their own methodology and a code in Matlab that they used for solving several 2D and simple 3D tasks [9], [10]. This paper presents the solution of a fully 3D example.

2. CONTINUOUS MATHEMATICAL MODEL

Consider a system of n 3D charged metal bodies $\Omega_1,\ldots,\Omega_n$ placed in the air (Fig. 1). The bodies with potentials $\varphi_1,\ldots,\varphi_n$ carry charges $Q_1,\ldots,Q_n$. All of them may be subject to motion.

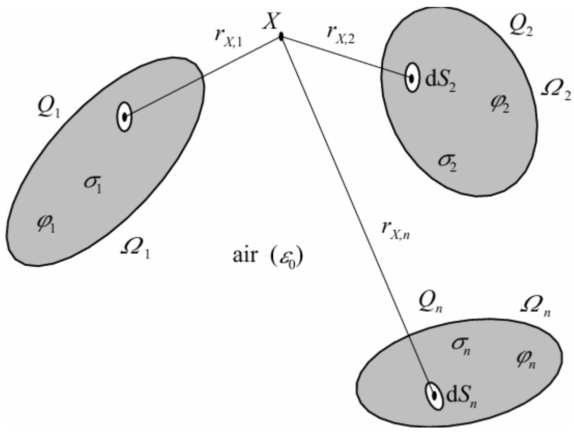

Fig. 1. A system of charged electrodes in the air

The task is to find at any given time instant:

- The distribution of charge on their surfaces.
- The distribution of electric field in the domain.
- The forces acting among the bodies.

First it is necessary to determine the electric potential φ at an arbitrary exterior or surface point X. The potential $\varphi(X)$ is given by relation

$$\varphi(X)=\frac{1}{4\pi\varepsilon_0}\cdot\sum_{i=1}^{n}\oint\!\!\!\oint_{S_i}\frac{\sigma_i\cdot\mathrm{d}S}{\left|r_X-r_i\right|}+\varphi_0 \qquad (1)$$

where the symbol S_i denotes the boundary of body Ω_i with surface charge density σ_i, $i=1,\ldots,n$. Further, $\left|r_X-r_i\right|$ is the distance between point X and integration point of the $i-$th surface and finally φ_0 is a constant.

In the next step we will find the charge distribution on the surfaces of all bodies $\Omega_1,\ldots,\Omega_n$. Using the fact that the surface of any perfectly electrically conductive body is an equipotential area, for any point of $X_j \notin S_j$ we obtain a system of the first-kind Fredholm integral equations in the form

$$\varphi(X_j)=\varphi_j=\frac{1}{4\pi\varepsilon_0}\cdot\sum_{i=1}^{n}\oint\!\!\!\oint_{S_i}\frac{\sigma_i\cdot\mathrm{d}S}{\left|r_{X_j}-r_i\right|}+\varphi_0, \qquad (2)$$
$$j=1,2,\ldots,n.$$

As far as the whole system is electrically neutral, it can be supplemented with the conditions

$$\oint\!\!\!\oint_{S_i}\sigma_i\cdot\mathrm{d}S=Q_i,\ i=1,\ldots,n \qquad (3)$$

and

$$\sum_{i=1}^{n}Q_i=0\,. \qquad (4)$$

After computation of the distribution of charge density σ_i, $i=1,\ldots,n$ over the surfaces of all charged bodies we obtain the complete tool for evaluating the potential φ at every point in the investigated domain.

Finally, the Coulomb force acting on the body Ω_j is

$$\boldsymbol{F}_j=\frac{1}{4\pi\varepsilon_0}\cdot\sum_{i=1}^{n}\oint\!\!\!\oint_{S_j}\oint\!\!\!\oint_{S_i}\frac{\sigma_j\sigma_i}{r_{j,i}^3}\cdot\boldsymbol{r}_{j,i}\,\mathrm{d}S_i\mathrm{d}S_j, \qquad (5)$$
$$i,j=1,\ldots,n.$$

3. DISCRETE MATHEMATICAL MODEL

This paragraph is devoted to the methodology of the numerical processing of (1) and (2) without preliminary regularization. In this case the first step of the algorithm is the discretization of all surfaces $S_1,\ldots,S_n$. The best way is to use suitable triangular meshes that are able to appropriately cover (with a small error) almost every surface. Then it is necessary to suggest the distribution of the charge density inside every element. As mentioned before, the simplest approach is to consider it there constant. The assembly of the system matrix is then very fast, but at the expense of a high number of DOFs and smaller accuracy.

Generally, the distribution of charge density in the $l-$th element of the $k-$th surface may be expressed as

$$\sigma(Q_{k,l})\doteq\sum_{t=1}^{s_{k,l}}a_{k,l,t}\cdot f_{k,l,t}(x,y,z). \qquad (6)$$

Here $a_{k,l,t}$ are the coefficients and $f_{k,l,t}(x,y,z)$ are the partial testing functions. Symbol $s_{k,l}$ denotes the selected number of approximation terms in the corresponding element. With respect to further operations with these approximations it is advantageous when these functions satisfy the condition of orthogonality, i.e.

$$\int_{\Omega_{kl}}f_{k,l,u}(x,y,z)\cdot f_{k,l,v}(x,y,z)\cdot\mathrm{d}\Omega=0\quad\text{for}\quad u\neq v,$$
$$\int_{\Omega_{kl}}f_{k,l,u}(x,y,z)\cdot f_{k,l,v}(x,y,z)\cdot\mathrm{d}\Omega>0\quad\text{for}\quad u=v,$$
$$u,v=1,\ldots,s_{k,l}. \qquad (7)$$

These approximations have to be substituted into (2) and (3). Now the system (2) must gradually be multiplied by particular testing functions used in the cell with index kl and integrated over its surface $S_{k,l}$. The result is a system of linear equations providing the coefficients $a_{k,l,t}$ in particular cells.

4. ILLUSTRATIVE EXAMPLE

The methodology is demonstrated on the computation of time-variable electric field of a disconnector. The disconnector is depicted in Fig. 2

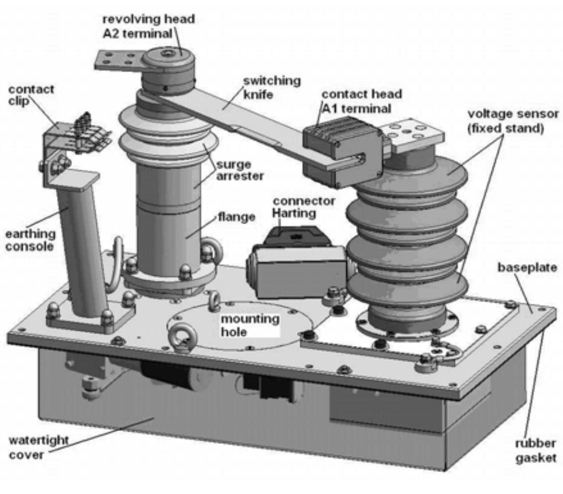

Fig. 2. Full view of the disconnector

The device has two contacts. While the fixed contact is represented by a terminal (in the right upper part), the movable contact is represented by the switching knife (in the middle on the top). For the calculation we used a substantially simplified arrangement according to Fig. 3. This figure also contains the principal dimensions of the contacts.

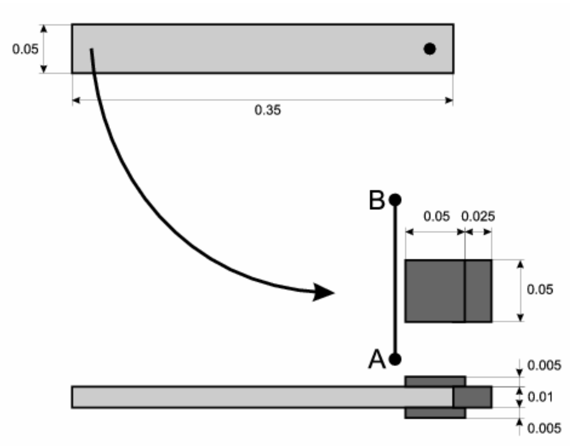

Fig. 3. Details of both contacts of the disconnector

The potential of the fixed contact is 0 V while the potential of the knife is 1400 V. The computations were carried out using the zero-order method.

The distribution of electric field (potential and electric field strength) was calculated for every 15°. The discretization mesh contained about 6000 elements, some of them can be seen in Fig. 4 showing the distribution of the potential in the plane of symmetry in the proximity of both contacts for angle 75° (for 90° both contacts are connected by the knife).

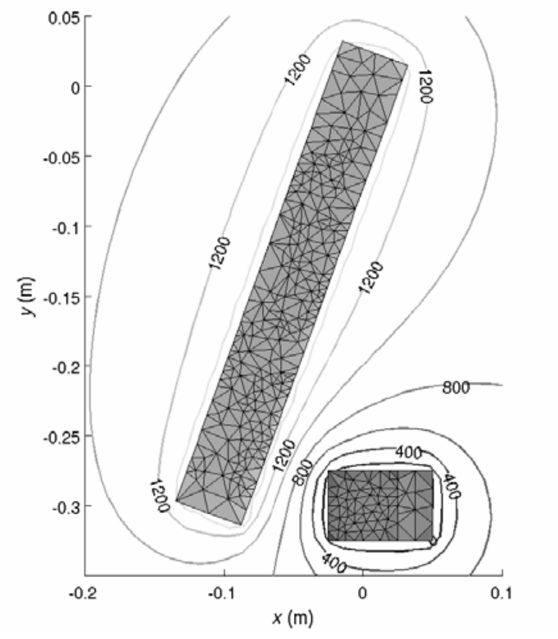

Fig. 4. Distribution of the electric potential in the domain of both contacts for angle 75°

Fig. 5 depicts the distribution of the module of electric field strength $|\boldsymbol{E}|$ along the line AB (see Fig. 3) for particular positions of the knife differing by defined angles.

In order to validate the results obtained, the same 3D arrangement was also calculated by the finite element method using the code COMSOL. Unfortunately, we could not use a sufficiently dense 3D mesh because we were limited by about 800000 elements (computer memory).

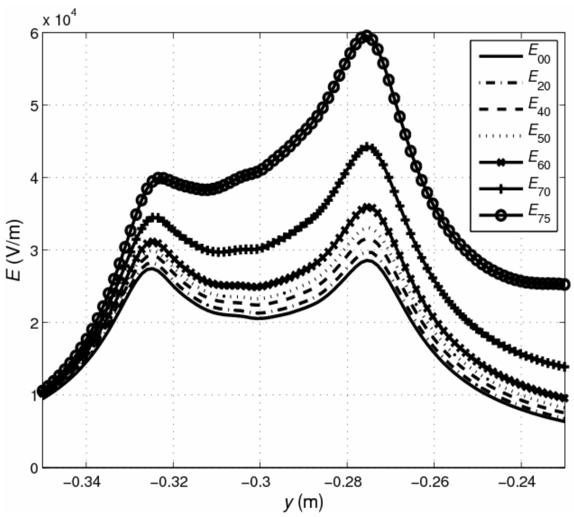

Fig. 5. Distribution of the module of electric field strength $|\boldsymbol{E}|$ along line AB (see Fig. 4) for different positions of the knife

The results are of the same character (see Fig. 6), and we were not able to decide which of them is more accurate. The principal reason is that our possibilities of verifying the convergence of the results on the fineness of the discretization mesh were rather limited by the range of the problem.

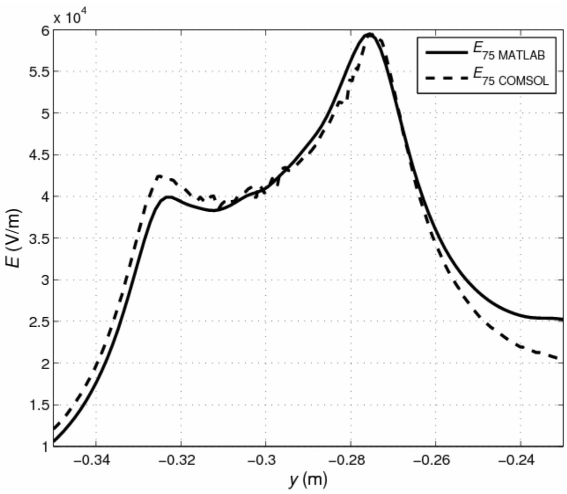

Fig. 6. Comparison of the results for one selected position of the knife (integral method versus finite element method)

5. CONCLUSION

The integral method may represent quite a powerful tool for investigating 3D large electrostatic fields. The time of computation is comparable or even shorter due to the number of mesh elements that is approximately by two orders lower than in case of FEM. Nevertheless, the results should be always validated by a suitable experiment or another reliable method of computation.

Acknowledgement

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